

Introduction for Carlo's Special Issue

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Back in 1994, when I started my PhD on the Boltzmann equation and asked my advisor how I should get acquainted with it, he told me that I just had to read Carlo Cercignani's treatise, *The Boltzmann equation and its applications*. Only years later would I realize the truly incredible work made by the author to compile and organize results borrowed from hundreds of references that were scattered in a variety of sources in mathematics, mechanics, statistical physics, numerical simulations, etc.—all with just one common point: the Boltzmann equation.

This book is just one example of Carlo's amazing productivity, with nearly three hundreds research papers published and ten books or so. Almost all of these show Carlo's passion for the Boltzmann equation, and more generally everything related to Boltzmann. Paradoxically, this focussing comes with an incredible diversity of methods, techniques and points of view. In 45 years of research, the Boltzmann equation led Carlo to work in theoretical mechanics, partial differential equations, numerical analysis, semigroup theory, spectral theory, Riemann-Hilbert problems, Fourier analysis, and many other areas. A few years ago, a collaboration with Sasha Bobylev on self-similar solutions of the Boltzmann equation even led him to a new pretty formula for the inversion of the Laplace transform. In brief, Carlo is at the same time one of the most focussed and one of the most versatile scientists that I know, defying any attempt of classification.

This productivity and versatility of his are not restricted to science, since a complete edition of Carlo's works should also include, among other things, a collection of poems, a comedy, translations of Homer, and a guide for helping Italians to pronounce Japanese. In fact Carlo would certainly fit amidst these intellectuals of the old times, who seem to know everything about science, literature, history, and speak an indecent number of languages.

The papers gathered in this special issue reflect the diversity of the areas of kinetic theory that Carlo has explored. Let me give some more details about three fields in which he was a pioneer.

The first one is the study of evaporation and condensation by means of the Boltzmann equation. In this approach, one considers the Boltzmann equation in a half-space (to be thought of as the blow-up of a transition layer between gas

and liquid, say), with given hydrodynamic (Maxwellian) data at infinity. A typical question is to rigorously establish a phase transition diagram determining the conditions of existence of a stationary solution, in terms of some basic hydrodynamic quantities such as the velocity at infinity. At the beginning of the eighties, Carlo made important contributions to the field, including a conjecture about the classification of linearized stationary solutions in a half-space. It did not take long before some other mathematicians could rigorously prove his conjecture, and since then the area of half-space problems has undergone remarkable developments, accurately surveyed by the paper of Bardos, Golse and Sone in the present volume.

Another area to mention is the kinetic modelling of granular media, one of the currently most dynamic areas of collisional kinetic theory, to the point that it fills up the entirety of part II of this special issue. Inelastic variants of the Boltzmann and Enskog equations have been playing a crucial role in the modelling of granular materials, both in theory and applications. Here again, Carlo was one of the first to formulate the problems in a mathematically consistent way, with an influential review paper from 1995. Since then he has made, together with various collaborators (Sasha Bobylev, Irene Gamba and others), decisive contributions to the field, most of them published in the *Journal of Statistical Physics* in recent years. My own review article on the subject presents some of the most mathematical parts of the kinetic theory of granular media, which has led and will certainly lead again to fascinating problems.

The last topic which I shall evoke here is about modelling and the derivation of the Boltzmann equation from microscopic models. Several papers in part III will present current research results about this general problem, either in a classical or in a quantum context. In fact, the derivation of irreversible macroscopic equations from microscopic models is nowadays one of the most basic problems in statistical mechanics. But it is worth recalling that until the end of the sixties, many physicists (and mathematicians) strongly doubted that Boltzmann's approximation could be put on a rigorous basis and that a nonlinear collisional kinetic model could be derived from Newton's laws of dynamics. Carlo cherished the Boltzmann equation too much to accept this point of view. In 1972, elaborating on earlier ideas by Grad, he wrote a short but conceptually important paper demonstrating the mathematical consistency of the derivation of the Boltzmann equation from the dynamics of a system of many hard spheres. Then the transition from microscopic to macroscopic dynamics was no longer a hand-waving argument based on physical intuition, but a *plausible* mathematical theorem, that could be proven if certain difficult but reasonable a priori estimates on the solutions could be established. This result was a premonition of Lanford's celebrated theorem, which appeared shortly after.

A common feature emerges from these three examples: Carlo's ability to translate problems coming from modelling and physics, into precise formulations that are intelligible to mathematicians. By doing so, he has given work to a whole

mathematical community, for the past four decades and probably for many years to come.

Hundred years after the tragic death of Ludwig Boltzmann, his scientific inheritance still lives on, and research around the Boltzmann equation has never been so active as it is now. Carlo is one of those who have most importantly contributed to this. All those—including me in the first place—who love the Boltzmann equation can be grateful to him.

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